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APPLICATIONS OF BRUNT'S RADIATION EQUATION TO MINIMUM TEMPERATURE FORECASTING

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During the last quarter century numerous studies have been directed toward the development of methods for forecasting minimum temperatures by the use of empirical mathematical formulae. While many such formulae have been suggested, it appears to the writer that they are quite generally identical in principle and differ from one another only with respect to the general form of equation used to represent the minimum temperature as some function of preceding temperature and/or hygrometric. This paper presents the results of applications of an entirely new form of radiation equation to minimum-temperature forecasting in two southern California fruit-frost districts and compares the values obtained with those derived through empirical formulae already in use in these districts.

The minimum temperature formula in general use today throughout all fruit-frost districts on the Pacific coast was developed by Young (1) and takes the general form:

$$T=d-\frac{h-n}{4}+V_d+V_h \tag{1}$$

where d and h are the 4:40 p. m. dewpoint and relative humidity; V_d and V_h are corrections which depend upon dewpoint and relative humidity; and n is a number which varies between districts and with cloudiness. For both the Riverside and El Centro key stations used in this study, n takes the value of 30 for clear or partly cloudy nights and 35 for cloudy nights.

It is not the purpose of this paper to analyze the various types of empirical minimum-temperature formulae; this has been accomplished by Ellison (2), who also presents a quite complete bibliography of the subject up to 1928. An examination of equation (1), however, reveals that the only variables in the formula are dewpoint and relative humidity. Since it is built up from hygrometric and minimum temperature data plotted together on a dot chart, other factors which affect the rate of temperature fall are indirectly included and it is not possible to subsequently remove these factors singly for analysis.

While these empirical formulae appear to succeed exceedingly well in areas where wind is lacking and low clouds are of infrequent occurrence on frost nights, they are not so successful in windy or cloudy districts.

According to Brunt (3, p. 125), "* * * the net outward radiation at night from the earth's surface depends only on atmospheric conditions and on the temperature of the earth's surface." Nevertheless, the temperature changes in the earth's surface produced by a given amount of radiation depends upon the rate at which heat is conducted to the surface from below to replace that lost through radiation. This rate, in turn, depends upon the density (π_1), specific heat (c_1), and the specific conductivity of heat (κ_1) of the soil surface. Values for density and specific heat of various types of soils are given in tables 1 to 3. Johnson and Davies (4) have computed values of κ_1, but give results for one type of soil only. If soil thermographs are available, however, this coefficient

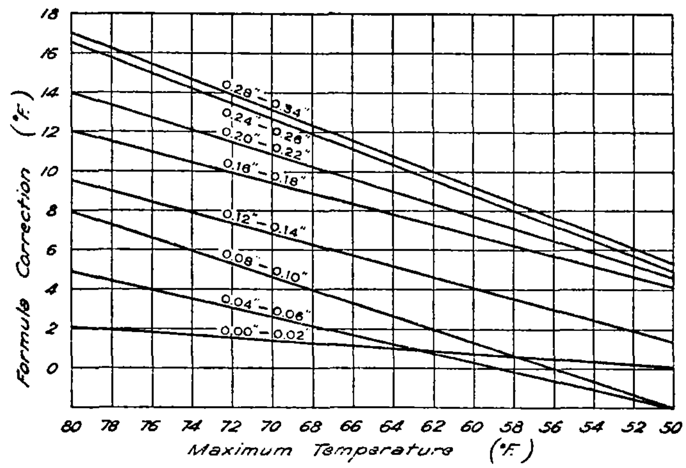

FIGURE 1.—Minimum-temperature-formula correction based on maximum temperature and pressure differences between Tonopah and Los Angeles.

may be computed (4) for any soil from measurements of lag in time of occurrence of maximum temperature at two depths by using the following expression:

$$\kappa_1=\frac{t\ (z_2-z_1)^2}{4L^2\pi} \tag{2}$$

where t is 24 (hours), z_2 and z_1 are two depths in the soil, and L is the lag in time (seconds) of occurrence of maximum temperature at depths z_1 and z_2.

TABLE 1.—*Specific gravities of composite soil separates: Whitney*[1]

Conventional name	Particle diameter	Specific gravity
	(mm)	
Fine gravel	2 -1	2.647
Coarse sand	1 -.5	2.655
Medium sand	.5 -.25	2.648
Fine sand	.25 -.10	2.659
Very fine sand	.1 -.05	2.680
Silt	.050-.005	2.698
Clay	.005-.000	2.837

[1] *Soils*, Lyon and Fippin, Macmillan Co., N. Y. 1912. 5th Ed. p. 95.

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TABLE 2.—*Specific heat of soils (equal weights)* [1]

Soil type	Specific heat	Soil type	Specific heat
Norfolk sand	0.1848	Podunk fine sandy loam	0.1828
Hudson River sand	.1769	Leonardtown silt loam	.1944
Fine sand (soil separate)	.1799	Hagerstown loam	.1911
Fine quartz flour	.1900	Galveston clay	.2097
Coarse sand (quartz)	.1900	Muck soil (25 percent organic matter)	.1566

[1] Patten, H.E., Bulletin 59, Bureau of Soils, U.S.D.A. 1909, p. 34.

TABLE 3.—*Effect of moisture on the specific heat of Podunk fine sandy loam* [1]

Moisture content (percent of dry weight)	Specific heat	Moisture content (percent of dry weight)	Specific heat
0.268	0.1850	6.60	.2324
1.33	.1935	10.08	.2575
2.14	.2000	20.25	.3204
2.83	.2053	26.93	.3562

[1] Specific Heats of Soils: Patten, H. E., Bulletin 59, Bureau of Soils, U. S. D. A.

Brunt concludes that the net loss of heat by radiation from the ground, R_N, is:

$$R_N=\kappa_1\rho_1c_1\left(\frac{\partial T}{\partial z}\right)_{z=0} \tag{3}$$

where z is the depth, and if R_N is assumed to be constant, then the temperature at $z=0$ is given by:

$$T_1=T_0-\frac{2}{\sqrt{\pi}}\frac{R_N}{\rho_1c_1\sqrt{\kappa_1}}\sqrt{t} \tag{4}$$

where t is the time in hours.

Brunt shows further that data on radiation from the sky obtained by various investigators may be very accurately represented by the formula:

$$R/\sigma T^4=a+b\sqrt{e} \tag{5}$$ [1]

where R is the measured radiation, σ is Stefan's constant (8.22×10^{-11}), $\sigma\ T^4$ is the total black body radiation at the surface temperature T, and e is the vapor pressure at the surface (in *mb*). He obtains somewhat different values for a and b in the cases of various sets of data but suggests that instrumental causes probably account for these differences.

In view of the fact that the fall in temperature during the night is only a small part of T, it may be assumed as a first approximation that R_N is constant, and since:

$$R_N=\sigma T^4(1-a-b\sqrt{e}) \tag{6}$$

then, by combining equations (4) and (6), he obtains:

$$T_1=T_0-\frac{2\sigma T^4}{\sqrt{\pi}}\left(\frac{1-a-b\sqrt{e}}{\rho_1c_1\sqrt{\kappa_1}}\right)\sqrt{t} \tag{7}$$

where T_0 is the air temperature at sunset.

By assigning values for ρ_1, c_1, and κ_1 (tables 1 to 5), appropriate for the soil conditions at the Riverside key station, and taking the mean of the values for a and b given in table 6, and assuming T constant at 280°, then equation (7) for the Riverside station becomes:

$$T_1=T_0-12.1\ (0.56-0.08\ \sqrt{e})\ \sqrt{t} \tag{8}$$

where T_0 is the 4:40 p. m. air temperature. By taking t constant at 13.5 hours,[2] equation (8) may be further simplified to:

$$T_1=T_0-44.5\ (0.56-0.08\ \sqrt{e}) \tag{9}$$

Using the appropriate soil constants, equation (7) for the El Centro key station becomes:

$$T_1=T_0-45.24\ (0.56-0.08\ \sqrt{e}) \tag{10}$$

TABLE 4.—*Specific conductivity of heat of soils* [1]

[Calculation of κ from time lag]

Z_1 (cms.)	Z_2 (cms.)	L (hours)	κ from eqn. 1
2.54	7.5	1.55	5.7×10^{-3}
2.54	15.25	4.5	4.2×10^{-3}
2.54	30.5	10.1	4.1×10^{-3}
		Mean [2]	4.7×10^{-3}

[1] Johnson and Davies. Q. J. Roy. Met. Soc. 33, 22, 1927.
[2] In one series during a period without rainfall, they found the value of κ to be 0.0035 and for a series during a period with heavy rainfall, a value of 0.0041, a difference of 0.0006. This difference is due to the fact that the water which replaces the air in the soil has a higher conductivity than the air.

TABLE 5.—*Relative conductivity of soil material* [1]

Soil material	Dry		Wet
	Loose	Compact	
Quartz powder	100.0	106.7	201.7
Peat	90.7	90.7	94.3
Kaolin	90.7	96.4	155.6
Chalk	85.2	92.6	153.2
Clay with limestone stones	112.1		
Clay with quartz stones	115.6		
Quartz sand (fine)	100.0		
Quartz sand (medium)	103.6		
Quartz sand (coarse)	105.3		
Quartz sand	100.0	174.0	189.0

[1] Relative Conductivity of Soil Material; Pott, H. E., Landw. Versuchs-Stationen, XX, p. 288.

TABLE 6.—*Values of a and b determined by various investigators* [1]

	a	b	Range of e (mb.)
Dines (Benson)	0.52	0.65	7–14
Asklöf (Upsala)	.43	.082	2–8
Angstrom (Bassour)	.48	.058	5–15
Boutaric (France)	.60	.042	3–11
Robitsch (Lindenberg)	.34	.110	3–22
Ramanathan and Desai (Poona)	.26	.120	8–18
Mean	.44	.080	

[1] Brunt, D., Physical and Dynamical Meteorology, 1934, p. 124.

The results of applying Brunt's equation to minimum-temperature forecasting on 25 clear nights at the Riverside station when temperatures fell to 34° F or lower at some station in the Corona district, are given in table 7. This station exemplifies excellent radiation conditions and the empirical minimum temperature formula is quite successful. It may be observed that the minimum temperature computed after Brunt averages too low for the observed minimum temperatures at this station. This is the expected result when it is considered that the effects of wind on retarding the temperature fall by mixing the surface layers of air, are not taken into consideration by this equation, whereas the empirical formula in use at the station tends to raise the formula estimate in cases of low vapor pressures, which are, in these districts, generally indicative of wind.

[1] Equation (5) should be regarded as entirely empirical with the strictly theoretical justification in doubt.

[2] A difference of 1 hour for t in equation (8) (between $t=13$ and $t=14$) would result in an error of only 0.6° C in the formula estimate at the mean vapor pressure for this station.

It may also be observed that the average departure of the computed minimum temperature from the observed is greater in the case of Brunt's equation than in the case of the empirical formula. Since the empirical formula, however, is built up from mean conditions, it should be less erratic than the equation obtained by fitting Brunt's formula to the temperatures and vapor pressures at a particular hour.

TABLE 7.—*Computed and observed minimum temperatures for the Riverside key station on clear nights with minimum temperatures below 34° F. at some station in the district*

1937–38	4:40 p. m. air temperature	4:40 p. m. vapor pressure	Formula temperature (Brunt)	Formula temperature (Young)	Minimum temperature following a. m. in shelter	Formula temperature (Brunt) minus shelter temperature	Formula temperature (Young) minus shelter temperature	Wind (4:40 p. m.) Beaufort
	°C	Mb.	°C	°C	°C	°C	°C	
Nov. 26	15.6	10.1	2.0	1.7	2.2	−0.2	−0.5	Light.
27	17.2	10.1	3.6	2.8	1.7	+1.9	+1.1	Do.
Dec. 6	16.1	8.0	1.2	.6	−.6	+1.8	+1.2	Do.
7	15.0	8.0	.1	.0	.0	+.1	.0	Do.
14	12.8	10.5	−.6	.6	1.1	−1.7	−.5	Do.
15	13.3	10.9	.1	1.1	2.2	−2.1	−1.1	Do.
16	13.3	10.9	.1	1.1	2.2	−2.1	−1.1	Do.
18	19.4	5.3	2.7	.6	1.1	+1.6	−.5	Moderate.
20	12.8	2.9	−6.0	−1.1	1.1	−7.1	−2.2	Do.
21	11.7	5.8	−4.6	−2.2	−2.2	−2.4	.0	Light.
24	8.3	9.4	−5.7	−.6	−.6	−5.1	.0	Do.
Jan. 3	12.2	11.7	−.5	.0	1.7	−2.2	−1.7	Do.
4	16.7	6.9	1.1	−.6	.0	+1.1	−.6	Do.
5	16.7	6.9	1.1	−.6	−1.7	+2.8	+1.1	Do.
6	13.3	8.4	−1.3	1.1	.0	−1.3	+1.1	Do.
8	16.7	3.8	−1.3	−.6	.0	−1.3	−.6	Moderate.
22	15.6	9.4	1.6	1.7	.0	+1.6	+1.7	Light.
23	18.3	3.6	.1	−.6	.6	−.5	−1.2	Moderate.
24	14.4	3.8	−3.6	−1.1	−1.1	−2.5	.0	Gentle.
25	16.1	3.5	−2.1	−1.1	1.1	−3.2	−2.2	Moderate.
29	9.4	10.9	−3.8	.6	−1.1	−2.7	1.7	Light.
Feb. 1	11.1	6.9	−4.5	−.6	1.7	−6.2	−2.3	Do.
6	15.0	5.8	−1.3	−1.1	−1.1	−.2	.0	Do.
12	10.6	8.7	−3.8	−1.1	−1.1	−2.7	.0	Do.
13	10.6	8.0	−4.3	−1.7	.0	−4.3	−1.7	Do.
Mean	14.1	7.6	−1.2	0.	.3	[1] 2.3	[1] 1.0	
Mean (°F)	57.4	([2])	29.8	32.0	32.5	[1] 4.1	[1] 1.8	

[1] Average departure regardless of sign.
[2] Mean dewpoint 38° F.

Brunt's equation was also applied to data accumulated on 25 clear nights at the El Centro key station (table 8) where empirical formulae are not so successful. It was found that the formula estimate computed after Brunt tended to average slightly too high while the mean departure from the actual minimum temperature was quite large. It may be remarked in this connection that the 4:40 p. m. observation at El Centro is made in a grapefruit grove several miles from town and, as a result of transpiration from foliage and local irrigation in the vicinity of the station, vapor pressures average much too high to be representative of vapor pressures over the valley as a whole. This is evidenced by comparing the 4:40 p. m. dewpoints with those obtained at an urban station in El Centro at 7 p. m. The latter dewpoints are invariably lower than those obtained in the grove. For the data in table 8, the 7 p. m. dewpoints average 6° F. lower than those at 4:40 p. m.[3] with a maximum difference of 15.5° F. on January 23. This mean difference in dewpoints would lower Brunt's estimate from 1° C. to 2° C. It has not been deemed advisable to substitute these 7 p. m. vapor pressures in Brunt's equation, however, since the El Centro empirical formula has been constructed from data obtained at the grove station and it was desired to compare the two formulae by using identical data.

[3] This difference in mean dewpoints is too great to be assigned to purely diurnal effects

TABLE 8.—*Computed and observed minimum temperatures for the El Centro key station on clear nights with minimum temperatures below 32° F. at some station in the district*

1937–38	4:40 p. m. air temture	4:40 p. m. vapor pressure	Formula temperature (Brunt)	Formula temperature (Young)	Minimum temperature following a. m. In shelter	Formula temperature (Brunt) minus shelter temperature	Formula temperature (Young) minus shelter temperature	Wind (mean) Beaufort
	°C	Mb	°C	°C	°C	°C	°C	
Dec. 6	16.2	11.5	3.1	3.1	1.7	+1.4	+1.4	Light.
21	12.1	8.3	−2.8	.3	−.7	−2.1	+1.0	Calm.
Jan. 5	15.0	10.9	1.6	1.8	−.3	+1.9	+2.1	Do.
6	15.9	8.7	1.2	.8	.6	+.6	+.2	Do.
8	13.7	8.9	−.9	.9	.8	−1.7	+.1	Light.
9	17.3	5.0	.1	−1.2	−.1	+.2	−1.1	Do.
10	14.6	7.2	−1.0	−.8	−.4	−.6	−.4	Do.
11	16.2	9.6	2.1	1.8	.0	+2.1	+1.8	Calm.
12	17.9	8.7	3.2	1.6	.0	+3.2	+1.6	Do.
16	16.9	11.0	3.6	2.9	1.9	+1.7	+1.0	Do.
17	21.6	11.4	8.5	3.8	4.3	+4.2	−.5	Moderate.
20	13.6	6.2	−2.7	−1.7	6.3	−9.0	−8.0	Do.
21	17.9	6.4	1.7	−.1	.8	+.9	−.9	Light.
22	20.7	8.5	6.0	2.3	−.7	+6.7	+3.0	Calm.
23	18.7	8.9	4.0	2.1	−2.3	+6.3	+4.4	Do.
24	13.1	5.1	−4.0	−2.7	−1.9	−2.1	−.8	Light.
25	12.2	5.9	−4.4	−2.6	−.6	−3.8	−2.0	Do.
26	16.0	7.1	.2	−.3	−.1	+.3	−.2	Calm.
27	17.8	8.6	3.1	1.4	−.1	+3.2	+1.5	Do.
30	15.6	8.5	.9	.9	−.5	+1.4	+1.4	Do.
Feb. 1	15.7	15.2	4.5	4.3	1.8	+2.7	+2.5	Light.
2	16.1	9.0	1.6	1.2	4.9	−3.3	−3.7	Do.
4	18.2	8.4	3.4	1.4	5.5	−2.1	−4.1	Moderate.
13	17.1	5.9	.5	−1.1	5.6	−5.1	−6.7	Light.
17	10.6	6.4	−5.6	−2.2	−1.2	−4.4	−2.0	Calm.
Mean	16.0	8.4	+1.1	+0.7	+1.0	[1] 2.8	[1] 2.1	
Mean (°F)	60.8	([2])	34.0	33.3	33.8	[1] 5.0	[1] 3.8	

[1] Average departure regardless of sign.
[2] Mean dewpoint 40° F.

Values obtained by applying Brunt's equation (with T constant at 273°) to data collected at El Centro during the freeze of January, 1937, are presented in table 9. Evidently the vapor pressures obtained during this period when wind movement was fairly rapid and consistent, were more representative of conditions over the district since the computed minimum temperatures average considerably lower than the observed. These values are interesting in that they give some idea as to how low temperatures might have fallen in the El Centro district if factors such as wind, and the effects of general irrigation for frost protection, had not interfered with the temperature fall.

TABLE 9.—*Computed and observed minimum temperatures at El Centro for the freeze of January 1937*

1937	4:40 p. m. air temperature	4:40 p. m. vapor pressure	Formula temperature (Brunt)	Formula temperature (Young)	Actual minimum temperature following a. m. in shelter	Formula temperature (Brunt) minus shelter temperature	Formula temperature (Young) minus shelter temperature	Lowest temperature in district
	° C.	mb	° C.	° C.	° C.	° C.	° C.	° C.
Jan. 8	6.1	4.8	−9.5	−5.6	−6.0	−3.5	+0.4	−7.2
9	3.8	4.8	−11.8	−5.6	−5.0	−6.8	−.6	−6.7
10	7.3	5.1	−8.1	−4.4	−4.4	−3.7	.0	−5.6
21	4.4	1.6	−14.2	−9.4	−9.5	−4.7	+.1	−11.1
22	4.1	1.6	−14.5	−9.4	−7.2	−7.3	−2.2	−11.1
23	5.6	2.4	−10.9	−6.7	−7.8	−3.1	+1.1	−8.8
24	12.2	6.6	−2.2	−1.7	−5.2	+3.0	+3.5	−6.7
25	10.0	4.1	−6.1	−3.9	−5.0	−1.1	+1.1	−8.9
26	10.9	4.1	−5.2	−3.9	−4.4	−.8	+.5	−6.7
Mean	7.2	3.9	−9.2	−5.6	−6.1	[1] 3.8	[1] 1.1	−8.1
Mean: (° F.)	45.0	([2])	15.4	21.9	21.0	[1] 6.8	[1] 2.0	17.4

[1] Average departure regardless of sign.
[2] Mean dewpoint 22° F.

Since the net loss of heat by the earth's surface on cloudy nights is diminished by an amount approximately equal to the radiation from the base of the cloud, it should be possible to correct any minimum temperature formula for cloud effects if data regarding radiation from cloud surfaces or the effects of clouds on radiation from the earth, can be obtained. Asklöf (5) presents data on the net radiation from the ground on clear nights and on cloudy nights (table 10) and, while these data are somewhat meager, they may be utilized in this study until more complete observations concerning radiation on cloudy nights are available.

Ångström (6) computes that the net loss of radiation from the ground on cloudy nights is:

$$R_m-(1-0.09m)\,R_0 \tag{11}$$

where R_m is the net loss of radiation from the ground with m tenths of the sky covered by cloud, and R_0 is the net radiation on clear nights. Ångström's formula, however, obviously cannot deal with clouds of different heights.

TABLE 10.—*Values of K for various cloud heights computed from Asklöf's radiation data*

Cloud type	Average height (km)	Net radiation (Asklöf)[1] (gm=cal./cm²)	$K^1 \left(1-\frac{R_m}{R_0}\right)$
Nimbus, stratus or strato-cumulus	1.5	0.023	0.864
Alto-cumulus (alto-stratus)	2.8	.039	.769
Cirro-stratus	6.4	.135	.200
Clear sky	----------	.169	.000

[1] Geog. Annaler, Stockholm, 2, 1920, p. 253.

By taking Asklöf's radiation data contained in table 10, it is possible to compute a correction (K) for cloudiness to be applied to Brunt's equation such that:

$$K=1-\frac{R_m}{R_0} \tag{12}$$

where R_m is the net raidation on a totally cloudy night and R_0 is the net radiation on a clear night. Then if the temperature fall during the night, as computed by equation, is Δt, it follows that:

$$T_1=T_0-\Delta t(1-Km) \tag{13}$$

where m is the number of tenths of sky covered by cloud. By plotting the values for K given in table 10 as a function of cloud height, a curve is obtained which may be approximately represented by:

$$K=0.92-0.005h-0.017h^2 \tag{14}$$

where h is the height of the base of the clouds above the surface (in kilometers).

The results of applying the correction K to Brunt's estimate of the minimum temperature on 15 nights with clouds at the El Centro key station, are given in table 11.

On first analysis, Brunt's equation appears considerably more accurate than Young's formula for these data. It should be considered, however, that Young's formula has been constructed from data accumulated on cold nights [4] and is therefore intended to apply only on nights which are generally clear or, at most, on nights when clouds clear early in the forecast period. The cloud data contained in table 11 are for mean conditions throughout the night and while it is not usually possible to predict the actual height of clouds during the night or the mean percentage of sky covered by clouds with any great degree of accuracy, the order of magnitude of the temperature correction for varying cloud conditions is well illustrated in the table.

[4] For frost forecasting purposes, a cold night is defined as one on which the temperature falls to 32° F, or lower, at one or more stations in the fruit-frost district.

TABLE 11.—*Computed and observed minimum temperatures for the El Centro key station on a series of cloudy nights*

Date 1938	4:40 p. m. air temperature	4:40 p. m. vapor pressure	Formula temperature (Brunt)	Formula temperature (Young)	Minimum temperature following a. m. in shelter	Formula temperature (Brunt) minus shelter temperature	Formula temperature (Young) minus shelter temperature	Mean: Clouds: Amt.	Clouds: Type	Wind
	°C	mb	°C	°C	°C	°C	°C			
Jan. 8	12.1	9.8	−0.8	0.5	−0.7	−0.1	+5.7	0.1	ASt	Light.
18	21.1	9.7	10.7	4.1	9.9	+.8	−5.8	.3	StCu	Moderate.
19	16.7	8.3	8.5	.9	7.5	+1.0	−6.6	.5	StCu	Do.
29	13.1	9.8	.4	1.8	−.4	+.8	+2.2	.1	StCu	Calm.
31	13.6	6.6	8.6	2.4	7.7	+.9	−5.3	.8	StCu	Do.
Feb. 5	14.4	7.8	1.5	−.2	1.9	−.4	−2.1	.2	ACu	Light.
6	16.6	7.8	2.6	.8	.5	+2.1	+.3	.1	ACu	Do.
7	17.4	7.3	3.4	.4	3.3	+.1	−2.9	.5	CiSt	Calm.
8	15.5	10.6	6.2	1.9	6.7	−.5	−4.8	.4	ACu	Do.
9	16.6	9.5	4.9	1.7	4.3	+.6	−2.6	.2	StCu	Light.
10	21.0	12.8	9.6	4.9	4.9	+4.7	.0	.4	CiSt	Do.
11	21.8	9.0	12.3	2.7	11.1	+1.2	−8.4	.4	StCu	Moderate.
14	18.1	8.2	6.6	1.4	7.3	−.7	−5.9	.3	ASt	Do.
16	13.9	9.1	1.9	1.2	1.6	+.3	−.4	.2	StCu	Calm.
18	13.3	6.6	7.0	−1.3	6.7	+.3	−8.0	.7	St	Do.
Mean	16.3	8.9	5.6	1.5	4.8	[1] 1.0	[1] 4.1	.35		
Mean (°F.)	61.3	[2]	42.1	34.7	40.6	[1] 1.8	[1] 7.4	----		

[1] Average departure regardless of sign.
[2] Mean dewpoint 42° F.

The effects due to the mechanical action of night winds are difficult to determine in the ordinary minimum temperature formula. Such winds are of frequent occurrence in most districts and the tendency, in practically all cases, is to raise the surface temperature through mixing the warmer air aloft with that which is undergoing cooling at the surface. In extreme cases, the temperature inversion may be completely destroyed.

Since the total effect of a given wind velocity on the minimum temperature will depend upon the temperature gradient aloft at low levels, it becomes necessary to obtain some measure of temperatures at heights above the surface. Young (7) has shown that the vertical temperature gradient at low levels is largely a function of the maximum temperature during the day. Johnson (8), in an unpublished paper, has utilized this principle and presents the results of applying a correction to the minimum temperature at Corona, Calif., based upon pressure differences and maximum temperatures. He has taken the pressure difference between Tonopah, Nev., and Los Angeles as being representative of actual existing pressure gradients. It would probably be better to obtain gradients directly from the evening weather map and compute probable wind velocities for these data.

If probable wind velocities for the night can be determined, they should be substituted in figure 1[5] in place of pressure differences. Such corrections, however, cannot be determined until a long series of observations is available.

If meteorological conditions can be assumed to remain constant throughout the duration of the night, then all possible variable factors have been accounted for in the equations which have been presented in this paper. Any significant change in conditions, however, will result in

[5] Fig. 1 was built up from pressure and minimum temperature data accumulated at the Corona key station over a period of 5 years.

errors in any equation used to forecast the minimum temperature. Ångström (**9**) has introduced a value Δt_d into his minimum temperature formula which takes into account changes of air mass and conditions dependent upon the weather map. It appears to the writer, however, that it would be more logical to analyze the weather map for expected changes in conditions and modify the formula estimate on the basis of these expected changes. For example, it is entirely possible to compute minimum temperatures for air masses which are already present on the evening weather map but which have not yet invaded the region for which minimum temperature forecasts are to be prepared.

CONCLUSIONS

According to Krick (**10**), Brunt's equation should be more valuable in forecasting minimum temperatures than purely empirical formulae. Nevertheless, it is difficult to determine the proper soil constants with any degree of accuracy and, it its present status, it is far less accurate than the empirical formulae already in use. Since Brunt's equation assumes that the minimum temperature measured in a standard instrument shelter varies directly as the soil temperature in the immediate vicinity of the shelter, it is interesting to compare the extreme variability between temperatures measured in a standard fruit region shelter at the El Centro key station and those recorded by an aluminum-backed minimum thermometer placed directly on the ground approximately 10 feet from the same shelter. These data are presented in table 12. Obviously, Brunt's equation in its present form cannot present minimum temperatures with any greater degree of accuracy than this relationship.

TABLE 12.—*Comparison of minimum temperatures recorded in a standard fruit-region shelter at the El Centro key station to the minimum temperatures recorded by a thermometer exposed on the ground open to the sky*

Date	Minimum temperature in shelter [1]	Minimum temperature on ground [1]	Shelter temperature minus ground temperature	Mean		
				Clouds		Wind
1938	°C	°C	°C	*Amount*	*Type*	
Jan. 16	1.9	0.3	1.6	0		Calm.
17	4.3	2.4	1.9	0		Moderate.
18	9.9	5.3	4.6	0.3	StCu	Do.
19	7.5	2.9	4.6	.5	StCu	Do.
20	6.3	.2	6.1	0		Do.
21	.8	−.9	1.7	0		Light.
22	−.7	−3.4	2.7	0		Calm.
23	−2.3	−6.3	4.0	0		Do.
24	−1.9	−6.9	5.0	0		Light.
25	−.6	−5.1	4.5	0		Do.
26	−.1	−4.4	4.3	0		Calm.
27	−.1	−3.9	3.8	0		Do.
29	−.4	−4.6	4.2	.1	StCu	Do.
30	−.5	−4.7	4.2	0		Do.
31	7.7	4.1	3.6	.8	StCu	Do.
Feb. 1	1.8	−2.7	4.5	0		Light.
2	4.9	−1.4	6.3	0		Do.
4	5.5	0	5.5	0		Moderate.
5	1.9	0	1.9	.2	ACU	Light.
6	.5	−3.7	4.2	.1	ACU	Do.
7	3.3	.8	2.5	.5	CiSt	Calm.
8	6.7	2.9	3.8	.4	ACu	Do.
9	4.3	.8	3.5	.2	StCu	Light.
10	4.9	1.6	8.3	.4	CiSt	Do.
11	11.1	4.9	6.2	.4	StCu	Moderate.
13	5.6	−1.7	7.3	0		Light.
14	7.3	1.6	5.7	.3	ASt	Moderate.
16	1.6	−4.5	6.1	.2	StCu	Calm.
17	−1.2	−6.4	5.2	0		Do.
18	6.7	−.1	6.8	.7	St	Do.
Mean	3.2	−.1	4.3			
Mean (°F)	37.8	30.0	7.8			

[1] Minimum temperature occurs on the morning following the date listed.

Since such variabilities mentioned above, however, are due largely to wind (and clouds), it would be possible to prepare corrections for Brunt's equation depending upon wind, cloudiness, and other variable factors which are already indirectly considered in the empirical formulae, and its accuracy would no doubt be greatly increased.

In spite of the present inaccuracies of the equation for forecasting purposes, it is believed that its use is highly justified as a supplement to present minimum temperature formulae. It has also proved to be invaluable as a study device.

For example, Brunt's equation in its present form may be used in a qualitative manner to:

1. Give some idea of the time it will be necessary to light orchard heaters on frost nights.
2. Indicate the rapidity with which temperatures may fall during calm periods on nights with low vapor pressures.
3. Compute the order of magnitude of the correction for differences in the length of night.
4. Forecast the probable effects on minimum temperatures of changes in air mass with known vapor pressures and temperatures.
5. Determine the effects of changes in soil moisture and soil cover on the rate of temperature fall.
6. Forecast minimum temperatures for new frost stations where date necessary for the construction of empirical minimum temperature formulae have not yet been collected.

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